Influence of Interfacial Waves in Stratified Gas-Liquid Flows

Interfacial stresses are calculated from new measurements of liquid height and pressure drop for fully developed horizontal stratified flow. These are related to wave properties. An improved design method is suggested.

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Introduction

For low gas and liquid flows in a horizontal pipe, a stratified regime exists whereby the liquid moves along the bottom of the pipe and the gas concurrently above it. The most widely used method for predicting frictional pressure drop and liquid height (holdup) for this system is the analysis of Taitel and Dukler (1976) that uses separate force balances for the gas and for the liquid.

This approach, however, has limitations because of the assumption that drag at the interface, τ_i, is the same as for a flat surface and because the Blasius equation is used to calculate the drag of the wall on the liquid, τ_{WL}. Experiments were therefore performed to determine τ_i and τ_{WL}. These were conducted with horizontal pipelines having diameters of 2.52 and 9.53 cm and with liquid viscosities of 1 to 80 cp (mPa · s).

In a previous paper (Andritsos and Hanratty, 1986) we defined two types of interfacial waves:

1. Regular two-dimensional waves
2. Large-amplitude irregular waves associated with a Kelvin-Helmholtz instability.

The large-amplitude waves can cause large increases in τ_i. A goal of this paper is to use this information to interpret measurements of τ_i.

The Taitel-Dukler method is found to do a good job in predicting the liquid height and the frictional pressure drop. However, considerable improvement is possible if the influence of waves on τ_i is taken into account. Improvements obtained by using a better relation for τ_{WL} than the Blasius equation are not so large.

It is found that the interfacial friction factor, f_i, increases linearly with gas velocity at gas velocities larger than needed to initiate waves, and that the proportionality constant is insensitive to pipe diameter. The effects of liquid viscosity and liquid flow rate are found to be of secondary importance over the range of liquid viscosities studied, and can be taken into account by assuming that f_i is a function of the ratio of the height of the liquid to the pipe diameter, h/D. This influence of flow properties on f_i can be related to wave properties by assuming that the difference of f_i from the value for a smooth surface, f_G, is related to the ratio of the wave amplitude to the wavelength.

These results are used to suggest improved design relations for the frictional pressure drop and holdup.

Background Literature

The first widely used correlation for frictional pressure drop and holdup (the fraction of the pipe cross section occupied by liquid) for stratified flows was developed by Lockhart and Martinelli (1949). It has been recognized for some time (Baker, 1954; Hoogendoorn 1959), however, that this approach can overpredict pressure drops by as much as 100%.

In recent years improved correlations (Johannessen, 1972; Taitel and Dukler, 1976) for pressure drop and holdup have been obtained by considering the momentum balances for fully developed flow for the gas phase,

$$-A_G\left(\frac{dp}{dx}\right) - \tau_{WG}P_G - \tau_i S_i = 0 \qquad (1)$$

and for the liquid phase,

$$-A_L\left(\frac{dp}{dx}\right) - \tau_{WL}P_L + \tau_i S_i = 0 \qquad (2)$$

As shown in Figure 1, Eq. 1 represents a balance between the pressure forces on the gas space and the resisting stresses at the gas-solid boundary, τ_{WG}, and at the gas-liquid interface, τ_i. Equation 2 is a balance between the forces due to pressure, the drag of the gas on the liquid, and the resisting stress at the liquid-solid boundary, τ_{WL}. By developing relations between τ_i, τ_{WL}, τ_{WG}, and the flow variables, correlations for the pressure gradient can be obtained either from Eq. 1 or Eq. 2. By eliminating the pressure gradient between Eqs. 1 and 2 a relation for the liquid holdup, $A_L/(A_L + A_G)$, or for the height of the liquid layer, h/D, is obtained.

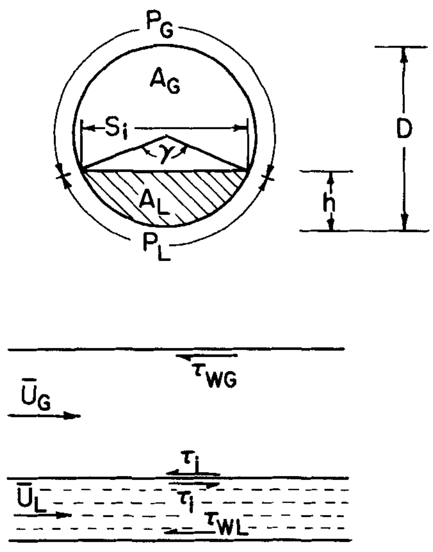

Figure 1. Flow system.

Taitel and Dukler (1976) represented the stresses in terms of friction factors

$$\tau_{WG} = f_G \frac{\rho_G U_G^2}{2} \quad \tau_{WL} = f_L \frac{\rho U_L^2}{2} \quad \tau_i = f_i \frac{\rho_G U_G^2}{2} \tag{3}$$

They suggested that $f_i = f_G$ and that f_G, f_L may be calculated by the Blasius equation or by Poiseuille's law where $Re_L = D_L U_L / \nu_L$ and $Re_G = D_G U_G / \nu_G$, with the hydraulic diameters defined as

$$D_L = \frac{4A_L}{P_L} \quad D_G = \frac{4A_G}{P_G + S_i} \tag{4}$$

This analysis provides a correlation similar to that suggested by Lockhart and Martinelli (1949) in that for a horizontal pipe the height ratio h/D and the Martinelli pressure drop parameters Φ_G and Φ_L are unique functions of the Martinelli flow parameter X. The Taitel-Dukler correlation is now widely used since it is in better agreement with pressure drop and holdup data for horizontal pipes than is the Martinelli relation and since it provides simple predictive methods for inclined pipelines.

Further improvements in this approach can be made by using more sophisticated correlations for f_i and f_L. In particular, the assumption that $f_i = f_G$ can cause large errors if waves exist at the interface. Under these conditions, τ_i is much larger than if the interface were smooth, and the Blasius equation does a poor job in predicting f_i. The use of the Blasius equation or Poiseuille's law to predict f_G and f_L appears to be less troublesome.

Russel et al. (1974) solved numerically the equation of motion for the liquid for the case of a laminar flow. Relations for the pressure drop and holdup obtained from this solution by assuming $\tau_i = \tau_G$ are only slightly different from those presented by Taitel and Dukler for the case of a turbulent gas-laminar liquid.

Cheremisinoff and Davis (1979) developed an equation for the velocity distribution in the liquid for a turbulent liquid flow. They assumed that τ_i does not vary with position on the interface and that stresses in the liquid are constant for a constant radial distance from the pipe center. An overall momentum balance gives the stress distribution in the liquid. The velocity gradient was related to the stress by using von Karman's eddy viscosity relation for $y^+ > 20$ and Deissler's eddy viscosity relation for $y^+ < 20$. The analysis thus predicts that the velocity varies only in the radial direction. Shoham and Taitel (1984) also used eddy viscosity concepts to develop a fully two-dimensional representation of the velocity field in the liquid. This was facilitated by developing an orthogonal transformation that maps the cross section occupied by the liquid onto a rectangular space.

More recently, a comprehensive study of the turbulent velocity field for stratified gas-liquid flow in a rectangular channel has been carried out at the Institut de Mecanique des Fluides de Toulouse (Suzanne, 1985) under conditions that waves exist at the interface. Of particular interest is the demonstration of the existence of secondary flows in the liquid, which can contribute to the transfer of momentum. This study suggests that the representation of the velocity field is more complicated than is implied by the turbulence analyses of previous investigators.

Cheremisinoff and Davis attempted to account for the effect of interfacial waves on f_i. They used measurements of Miya et al. (1971) to suggest that

$$f_i = 0.0142 \quad \text{for small-amplitude waves} \tag{5}$$

$$f_i = 0.008 + 2 \times 10^{-5}\, Re_L^* \quad \text{for roll waves} \tag{6}$$

where the modified Reynolds number is defined as

$$Re_L^* = \frac{U_{LS}}{\nu_L}\left(\frac{\pi D^2}{4S_i}\right) \tag{7}$$

The use of these relations provides good agreement with measurements of the pressure drop and holdup for air-water flow in a 2 in (50 mm) horizontal pipe. In contradiction to these results, a linear dependence of f_i on gas velocity is suggested from the works of Gayral et al. (1979) and of Hidy and Plate (1966).

Kowalski (1985) made direct measurements of the Reynolds shear stress in the gas for horizontal stratified flow in a pipe. Interfacial stresses obtained by an extrapolation of the Reynolds stress profile to the interface were found to be 15–30% lower than those calculated from a momentum balance. Kowalski showed that gas-to-wall friction factors are approximated quite well by the Blasius equation, provided the concept of a hydraulic diameter is utilized. He recommended the following equations for the interfacial friction factor for smooth and wavy surfaces:

$$f_i = 0.96\,(Re_G^+)^{-0.52} \tag{8}$$

$$f_i = 7.5 \times 10^{-5}(1-\alpha)^{-0.25}\, Re_G^{-0.3}\, Re_L^{0.83} \tag{9}$$

where α is the gas volume fraction and the Reynolds numbers are defined as

$$Re_G = \frac{U_G D}{\nu_G}, \; Re_L = \frac{U_L D}{\nu_L}, \; Re_G^+ = \frac{U_{GS} D}{\nu_G}$$

In this paper the Blasius equation is used to calculate τ_{WG}. Stresses τ_i and τ_{WL} can then be calculated from the measurements by using Eqs. 1 and 2. A slight improvement of the Blasius equation is suggested for estimating f_L. However, the principal contribution is in the development of a new correlation for f_i based on the recently reported study by Andritsos and Hanratty (1986) on the wave structure in gas-liquid stratified flows.

Analysis of the Measurements

Evaluation of τ_{WL} and τ_i

The stresses τ_{WL} and τ_i are calculated from Eqs. 1 and 2 from measurements of dp/dx and of h/D. The stresses τ_{WG} was calculated from Eq. 3 using

$$f_G = 0.046\ Re_G^{-0.2}. \qquad (10)$$

The hydraulic diameter of the gas space is calculated from Eq. 4 by using equations given by Govier and Aziz (1972, p. 563) to relate A_G, P_G, and S_I to the measured h/D. The use of this procedure to calculate τ_{WG} has support from direct measurements of the stress by Kowalski (1984) and from measurements of the pressure drop for single phase flows over two different inserts by Andritsos (1986).

Modified Taitel-Dukler correlation

The correlations developed by Taitel and Dukler for the pressure gradient and for h/D using the assumption that $f_i = f_G$ and the Blasius equation or Poiseuille's law to define f_i or f_L are shown in Figures 2 and 3. These correlations can be easily modified to account for values of $f_i \neq f_G$. The results of such a calculation are also shown in Figures 2 and 3 for different values of f_i/f_G.

Reasonable agreement is noted between the measured dp/dx and h/D and these modified Taitel-Dukler correlations. However, in some situations significant deviations were found that could be associated with errors (already noted by Andreussi and Persen, 1985) involved in using the Blasius equation to predict f_L.

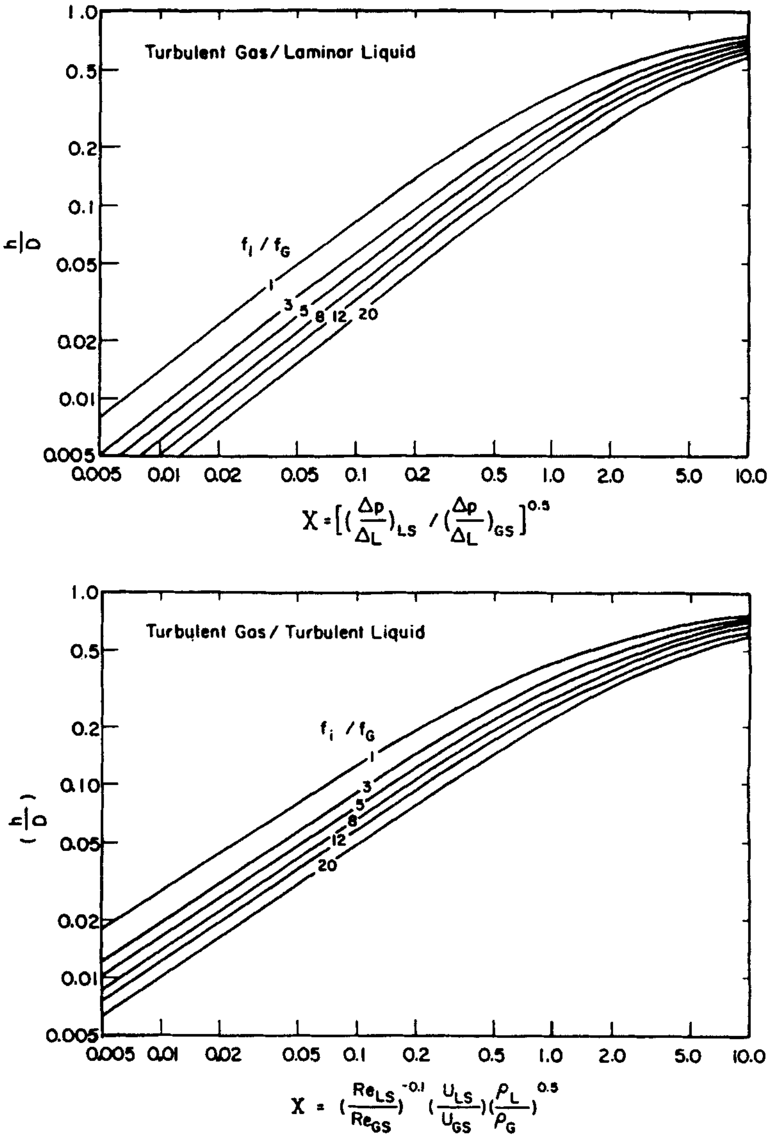

Figure 2. Film thickness as a function of f_i/f_G.

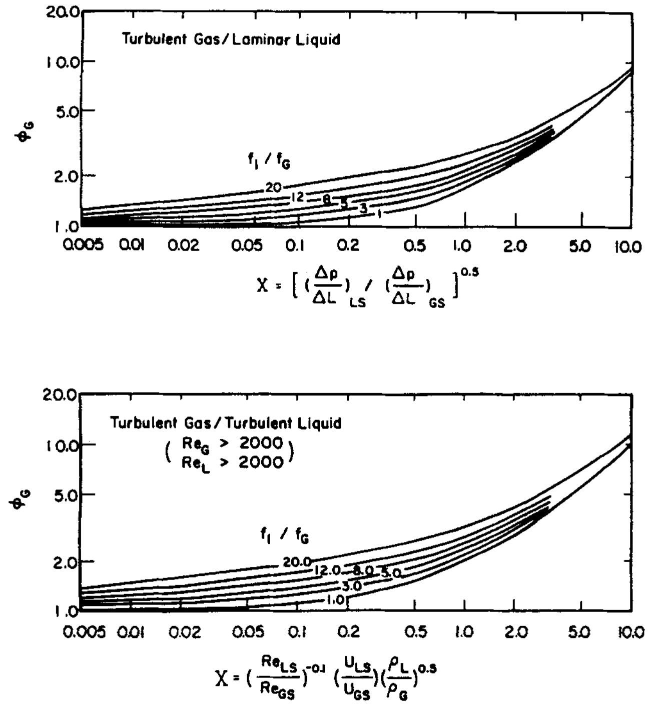

Figure 3. Dimensionless pressure drop as a function of f_i/f_G.

Modified Cheremisinoff-Davis correlation

Another way of dealing with the liquid phase friction is to develop a correlation for the dimensionless liquid height (Cheremisinoff and Davis, 1979). It is noted from the definition of the liquid phase friction factor f_L, given by Eq. 3, that

$$\frac{h^2(\tau_w/\rho)}{\nu^2} = \frac{1}{2} Re_L^2 \left(\frac{h}{D}\right)^2 \left(\frac{D}{D_L}\right)^2 f_L, \tag{11}$$

where D/D_L is a function h/D.

Cheremisinoff and Davis chose to develop a correlation for $h(\tau_w/\rho)^{1/2}/\nu$ rather than for f_L. The advantage is that such an approach is better able to account for changes in the shape of the velocity profile in the liquid caused by gas drag at the interface. The disadvantage, in comparison with the original Taitel-Dukler approach, is that a trial-and-error solution must be used to calculate the pressure drop and h/D from Eqs. 1 and 2, and the equations developed for f_i/f_G and $h(\tau_w/\rho)^{1/2}/\nu$.

A modification of the Cheresmisinoff-Davis correlation has been used in this research. The characteristic stress in the liquid is taken as

$$\tau_C = \frac{2}{3}\tau_{WL}\left(1 - \frac{h}{D}\right) + \frac{1}{3}\tau_i \tag{12}$$

A friction velocity and a dimensionless liquid height are defined as

$$u_c^* = (\tau_c/\rho_L)^{1/2} \tag{13}$$

$$h^+ = hu_c^*/\nu \tag{14}$$

As pointed out by Henstock and Hanratty (1976) in their treatment of annular flows this definition of a characteristic stress arises naturally from a consideration of laminar flows.

The analysis of laminar stratified flows by Russel et al. (1974) gives values of τ_{WL} that are 0 to 25% greater than calculated with Poiseuille's law. As was found by Henstock and Hanratty for annular flows, no effect of h/D and a very small effect of f_i/f_G is observed if the results are correlated by plotting h^+ vs. Re_L. The following equation is obtained for laminar flow:

$$h^+ = 1.082\, Re_L^{0.5} \tag{15}$$

An eddy viscosity approach is used to develop a relation for h^+ for turbulent flows. Following Cheremisinoff and Davis, the momentum balance in Eq. 16 gives the shear stress distribution in the liquid:

$$2(R - y)\theta\tau = R\gamma\tau_{WL} - 2\delta\tau_i - \frac{dp}{dx}[(A_L)_{r=R} - (A_L)_{r=R-y}] \tag{16}$$

The geometric variables defined in Figure 1 obey the following relations:

$$\frac{\delta}{D} = \left[\frac{h}{D} - \left(\frac{h}{D}\right)^2\right]^{1/2} - \left[\frac{h}{D} - \left(\frac{h}{D}\right)^2 - \frac{y}{D} + \left(\frac{y}{D}\right)^2\right]^{1/2} \tag{17}$$

$$\frac{y}{R} = 1 - \cos(\alpha/2)/\cos\theta \tag{18}$$

$$(A_L)_{r=R-y} = \frac{(R-y)^2}{2}(2\theta - \sin 2\theta) \tag{19}$$

Equation 16 assumes that τ_{WL} and τ_i are independent of θ and that τ depends only on radial distance from the wall y. Shoham and Taitel (1984) point out that the distance y becomes meaningless for $h/D > 0.5$. However, no turbulent liquid flow was observed for $h/D > 0.5$ for horizontal stratified flows.

The shear stress distribution in the liquid is related to the velocity gradient by using eddy viscosity concepts and the van Driest mixing-length relation:

$$\frac{\tau}{\tau_c} = \left(1 + \frac{\epsilon}{\nu_L}\right)\frac{du^+}{dy^+} \tag{20}$$

$$\frac{\tau}{\tau_c} = \left\{1 + \kappa^2 y^{+2}\left[1 - \exp\left(\frac{y^+}{26}\right)\right]\left|\frac{du^+}{dy^+}\right|\right\}\frac{du^+}{dy^+} \tag{21}$$

where $\kappa = 0.4$ is the von Karman constant. The velocity distribution in the liquid was obtained by integrating Eq. 21 for fixed values of h/D and f_i/f_G. This velocity distribution was then integrated over the cross section of the liquid to obtain the results shown in Figure 4. A small effect of h/D and a smaller effect of f_i/f_G is found. These calculated results can be represented by the equation.

$$h^+ = 0.098\, Re_L^{0.85} \Big/ \left(1 - \frac{h}{D}\right)^{0.5} \tag{22}$$

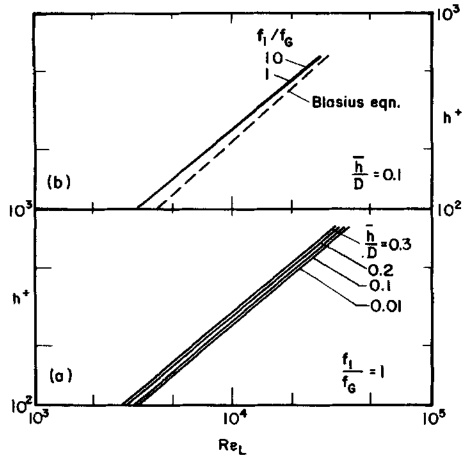

Figure 4. Effect of h/D, f_i/f_G on thickness relation.

A reasonable representation of the results for both laminar and turbulent flow is as follows:

$$h^+ = \left\{(1.082\ Re_L^{0.5})^5 + \left[0.098\ Re_L^{0.85} \Big/ \left(1 - \frac{h}{D}\right)^{0.5}\right]^5\right\}^{0.2} \quad (23)$$

Description of Experiments

The experiments were carried out in the horizontal gas-liquid flow loop described by Laurinat et al. (1984). Results were obtained in Plexiglas pipelines of 2.52 and 9.53 cm ID. Solutions of water and glycerine with viscosities of 1, 12, and 80 cp (mPa · s) were used in the 9.53 cm dia. pipe. Solutions with viscosities of 1, 4.5, 16, and 70 cp were used in the 2.52 cm dia. pipe.

The liquid height h and the wave velocity were measured using two parallel wire conductance probes that extended in the vertical direction over the entire cross section of the pipe. The wires were 0.51 mm dia. chromel; they were attached to the bottom of the pipe and were put under tension by a pair of weights. The separation was 0.25 cm for the 2.52 cm dia. pipe and 0.5 cm for the 9.53 cm dia. pipe.

The second probe was used to determine the wave velocity. It was located 26.7 cm downstream from the first probe in the 9.53 cm dia. pipe, 25.4 cm downstream in the 2.52 cm dia. pipe for the water runs, and 10.2 cm for the runs with viscous liquids. The upstream probe was located about 220 pipe diameters from the entry for the large pipe, 500 pipe diameters for water runs in the small pipe, and 350 diameters for the small pipe when used with the glycerine solutions. In the 9.53 cm dia. pipeline, in order to determine liquid hydraulic gradients at low gas velocities, measurements of the liquid thickness were taken with the parallel probes located 80 and 160 pipe diameters from the entry.

An oscillating signal was sent to the probes and an analyzer converted the response to an analog signal. A Wavetek model 182A 4 MHz function generator was used to provide the input sine signal at a frequency of 20 kHz. A demodulation circuit provided the peaks of the output signal synchronously using the square wave output of the function generator as a reference.

The output voltage was found to vary linearly with the conductance only at very high resistances ($R > 50{,}000\ \Omega$). A calibration of the analyzer was performed every day, to account for variations in the input voltage and room temperature. This calibration curve was fitted by a fifth-order polynomial equation in order to calculate liquid heights from conductivity measurements.

More details regarding the techniques used to measure liquid height can be found in theses by Lin (1986) and by Andritsos (1986).

Pressure drops were measured in the gas phase at low gas velocities with a Flow Corporation model MM3 micromanometer having a precision of ±0.01 mm of Meriam Red Oil (spg 0.827). At least three measurements were made for each run and the average value was used.

At high gas velocities the pressure drops in the liquid phase were measured by two diaphragm-type Viatran model 504 differential pressure transmitters. The range of the transmitters was 0–4 × 10^3 and 0–4 × 10^4 N/m^2. The pressure taps and the connection tubing were filled with liquid and frequently purged of gas bubbles. A hypodermic syringe was used to purge the air bubbles from the pressure taps for the runs with viscous solutions. Most of the pressure taps had a 1 mm dia., while a few had a 3.18 mm dia. No effect of tap diameter was found, except with the most viscous liquids, for which pressure taps of the same diameter had to be used in order to obtain steady readings.

The length over which the pressure drop was measured was varied depending on the flow rates. A distance of 16 m was necessary in order to get an appreciable pressure difference for a gas velocity of 1 m/s in the 9.53 cm dia. pipe.

To check the smoothness of the Plexiglas pipes, pressure differences were also taken for single-phase gas flows. These agreed reasonably well with the Blasius correlation, implying a hydraulically smooth pipe. Tests were also conducted to see if the alignment of the test sections caused any roughness problem. Measurements taken with the pressure taps in the same and in adjacent test sections revealed no indication of such a problem.

The use of the concept of hydraulic diameter for the gas phase in a smooth stratified flow was tested by measuring the pressure drop in a 2.52 cm dia. brass pipe with two interchangeable inserts. The two inserts were cut from a 2.54 cm dia. Plexiglas rod and were screwed in the bottom of the brass pipe. The ratios of the maximum height of the insert over the pipe diameter, B/D, were 0.207 and 0.511. The length of the brass pipe with the inserts was 3.05 m. It was connected downstream in the 2.52 cm dia. pipeline. Three pressure taps were located within the second half of the pipe. This allowed at least a 60 pipe diameter length before the pressure measurements were taken.

The results appear in Figure 5, where the Reynolds number is calculated using a hydraulic diameter. Reasonable agreement is noted with correlations for fully developed pipe flows, even for laminar flow.

Results

Interfacial stress

The principal focus of this research was to determine the interfacial stress, τ_i. This was calculated from Eq. 1, using measurements of dp/dx and h/D and the Blasius equation to evaluate τ_{WG}.

Measurements of f_i are plotted against the liquid Reynolds number in Figure 6. The dots and open triangles are, respectively, results obtained in the 2.52 cm dia. pipe and the 9.53 cm dia. pipe. The solid line is the Cheremisinoff-Davis correlation obtained from studies in a 5.08 cm dia. pipe. The filled triangles

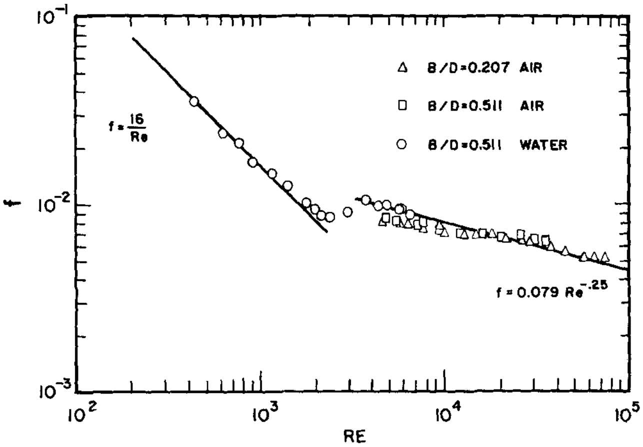

Figure 5. Pressure drop measurements for single-phase flow over two inserts.

are data obtained in a 5.08 cm dia. pipe by Laurinat (1979) in the University of Illinois facility. This plot shows that f_i does not correlate with Re_L, as suggested by Cheremisinoff and Davis. This disagreement is expected from the observations of Andritsos and Hanratty (1986) that transitions to various types of wave patterns are insensitive to changes of pipe diameter, if comparisons are made at the same gas velocity and h/D.

Figure 7 illustrates the strong effect of waves on f_i. Here it is shown that the increase of f_i over what would exist for a smooth interface correlates with the ratio of the amplitude of the wave, Δh, to the wavelength, λ.

Figures 8 and 9 illustrate the effect of gas velocity of f_i/f_G. For gas flow rates small enough (low U_{GS}) that no waves are generated at the interface, f_i/f_G was found to be close to unity. The effect when waves are present is different from what is found for air-liquid flow in a rectangular channel, for which no sudden change is observed in the value of the friction factor with a change from a three-dimensional wave pattern to a pattern with large amplitude roll waves (Hershman, 1956). In contrast, the pipe flows show a rapid increase in f_i/f_G with the appearance of the large-amplitude waves discussed by Andritsos and Hanratty. For experiments with liquids with a viscosity of 80 cp (mPa · s), for which no regular two-dimensional waves are observed, there is a sudden jump in f_i/f_G from a value of one to a value of about five, where the large-amplitude wave region is reached. Larger values of f_i/f_G are observed at low gas velocities in the

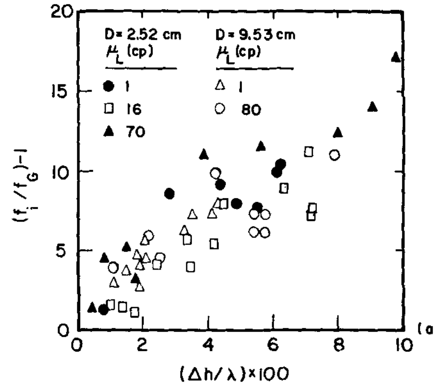

Figure 7. Effect of $\Delta h/\lambda$ on interfacial friction factor.

9.53 cm dia. pipe than in the 2.54 cm dia. pipe. This can be attributed to the existence of large two-dimensional waves with small ripples riding on them (Andritsos and Hanratty, 1986.)

Figures 8 and 9 show that the primary factor affecting f_i/f_G is the gas velocity, and that the relationship is approximately linear. The liquid rate and liquid viscosity are secondary effects. The influence of these variables appears to be represented by the dimensionless height, h/D, which increases with increases in either liquid viscosity or liquid flow rate. Figures 10 and 11 present values of f_i/f_G vs. the gas rate for three different ranges of h/D. For liquid viscosities greater than 12 cp only data in the wavy regimes are plotted. Considerable scatter is noted in all three plots. However, there is a clear effect of h/D, and it is worth noting that any specific trends with changes of liquid viscosity and pipe diameter are absent in these plots.

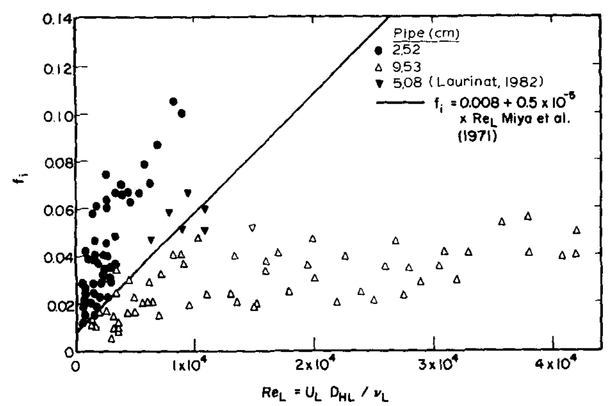

Figure 6. Interfacial friction factor as a function of Re_L.

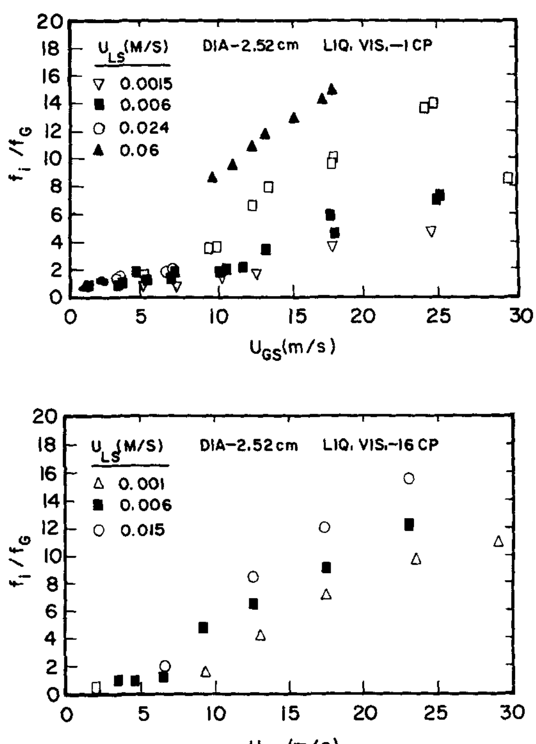

Figure 8. Effect of gas and liquid velocity on interfacial friction factor.

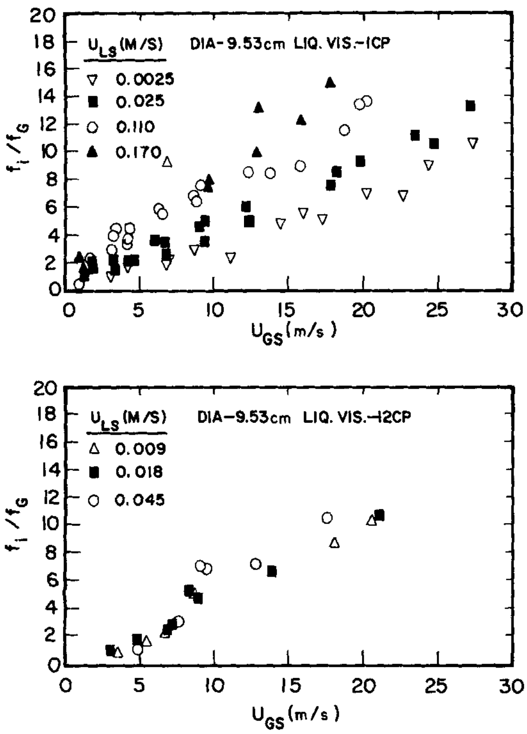

Figure 9. Effect of gas and liquid velocity on interfacial friction factor.

Liquid height

The values of τ_{WL} and τ_i calculated from the measurements of the pressure gradient and the liquid height can be used to evaluate the dimensionless height h^+, defined by Eqs. 12–14. Comparisons of measured h^+ with values calculated from Eq. 23 are given in Figure 12 for water flows in the 9.53 cm dia. pipe. At low gas velocities (small Re_L for a fixed U_{LS}) the measured h^+ are lower than predicted because the flows were not fully developed. At moderate gas velocities they are 0–40% higher than predicted. The same behavior, with greater scatter, is observed in Figure 13, where measured h^+ for the high-viscosity liquid are plotted.

However, even taking these discrepancies into account, Eq. 23 appears to give a good average fit to the measurements. In fact, this agreement is quite satisfying considering the complicated flow patterns that can exist in the liquid (Suzanne, 1985; Andreussi and Persen, 1985).

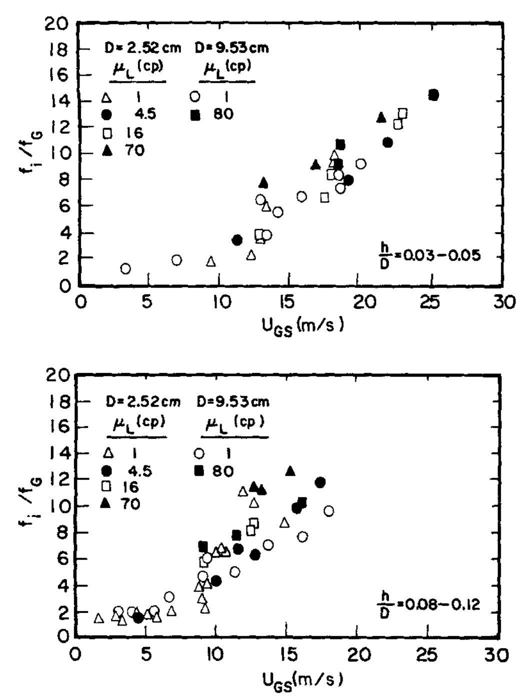

Figure 10. Effect of film thickness on interfacial friction factor.

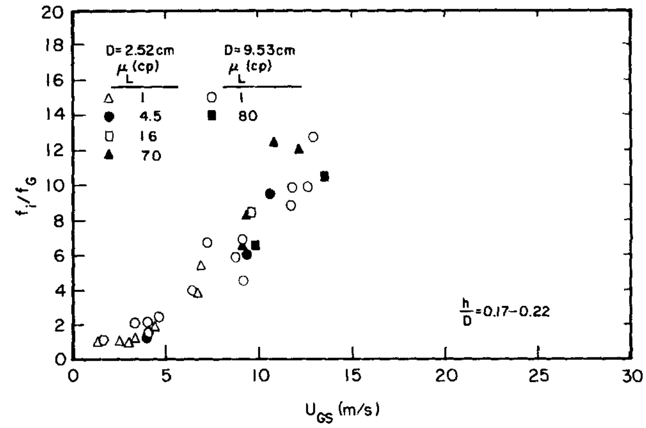

Figure 11. Effect of film thickness on interfacial friction factor.

Development of Design Relations

Correlation for f_i

As shown in Figure 7, f_i/f_G correlates quite well with $\Delta h/\lambda$. The utilization of this result to develop a relation of f_i/f_G with controlled variables is difficult because very little has been done

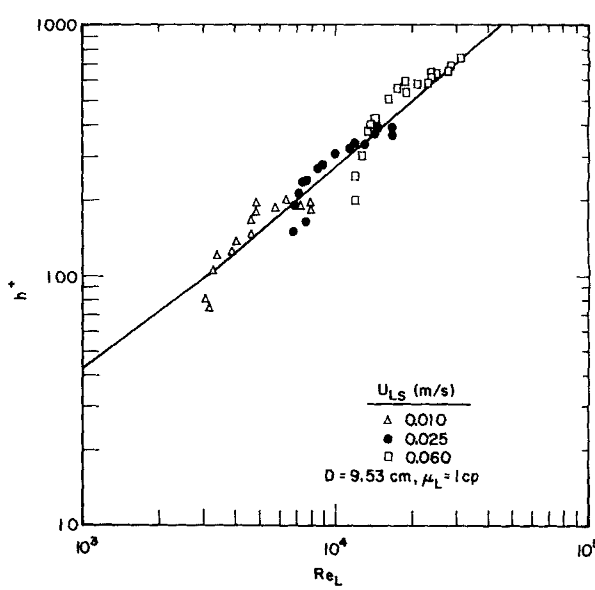

Figure 12. Comparison of experimental values of h^+ with theory.

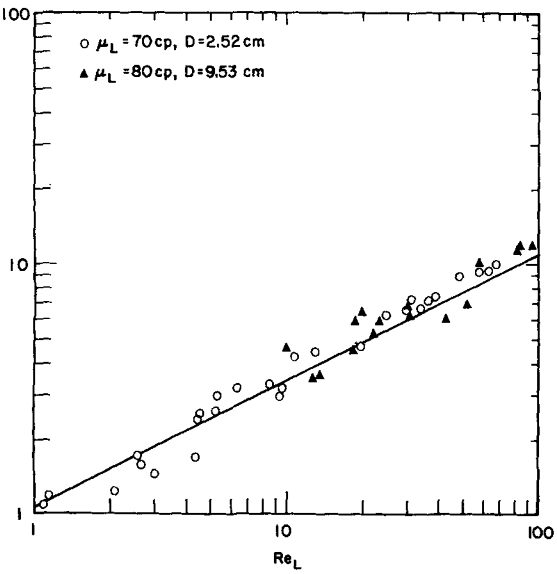

Figure 13. Comparison of experimental values of h^+ with theory.

to predict wave heights for concurrent gas-liquid flows. Saffman and Yuen (1982) have recently shown that

$$\frac{k}{g}C^2 + \frac{k\rho_G}{g\rho_L}(U_G - C)^2 - 1 = \frac{(2\pi)^2}{8}\left(\frac{\Delta h}{\lambda}\right)^2\left[\left(2\frac{k}{g}C^2 - 1\right)^2 + 1\right] \quad (24)$$

for an inviscid irrotational gas flow over an inviscid, irrotational stationary deep liquid. This equation gives

$$\frac{\Delta h}{\lambda} \sim \left[\frac{k}{g}C^2, \frac{k}{g}\frac{\rho_G}{\rho_L}(U_G - C)^2\right], \quad (25)$$

where k is the wave number, equal to $2\pi/\lambda$. Andritsos (1985) has observed that for stratified flows C will be fixed if U_G and h/D are fixed. This prompted him to suggest that

$$\frac{f_i}{f_G} - 1 \sim \left[\frac{h}{D}, U_{GS}\left(\frac{k}{g}\frac{\rho_G}{\rho_L}\right)^{1/2}\right], \quad (26)$$

where U_{GS} is the superficial gas velocity. This relation accounts implicitly for effects of liquid viscosity through h/D.

The experiments of Andritsos and Hanratty suggest that the characteristic wave number of the waves causing the increased drag is roughly equal to the minimum value, k_M, for which a Kelvin-Helmholtz instability is possible. For water, $k_M = 3.7$ cm^{-1}. Under the conditions of this investigation $(k_M/g)^{1/2}(\rho_G/\rho_L)^{1/2}$ is almost constant.

Andritsos and Hanratty have also shown that the gas velocity at which these large irregular waves appear, $U_{G,t}$, can be approximated by the equation

$$U_{G,t}\left(\frac{k_M}{g}\frac{\rho_G}{\rho_L}\right)^{0.5} = 1 \quad (26)$$

Consequently, Eq. 25 can be written as

$$\frac{f_i}{f_G} - 1 \sim \left(\frac{h}{D}, \frac{U_{GS}}{U_{GS,t}}\right) \quad (27)$$

The above kinds of considerations led Andritsos (1986) to propose the following empirical correlation of the measurements of f_i/f_G shown in Figures 8–11.

$$\frac{f_i}{f_G} = 1 \quad \text{for } U_{GS} \leq U_{GS,t} \quad (28)$$

$$\frac{f_i}{f_G} = 1 + 15\left(\frac{h}{D}\right)^{0.5}\left(\frac{U_{GS}}{U_{GS,t}} - 1\right) \quad (29)$$

A stability analysis for this system by Andritsos (1986) shows that $U_{GS,t}$ varies as $\rho_G^{-1/2}$. A rough estimate of $U_{GS,t}$ for atmospheric pressure is 5 m/s. A sufficiently accurate approximation of $U_{GS,t}$ for design calculations is as follows:

$$U_{GS,t} = 5\text{m/s}\left(\frac{\rho_{GO}}{\rho_G}\right)^{1/2} \quad (30)$$

where ρ_{GO} = the gas density at atmospheric pressure. An improved estimate of $U_{GS,t}$ could be obtained if the effects of U_{LS}, shown in Figure 8 and in the results of Kowalski (1985), were taken into account.

A comparison of measurements at atmospheric pressure with Eqs. 28–30 is given in Figure 14. This equation underpredicts the friction factor for water at low and high gas rates. It overpredicts the friction factors in a range of superficial gas velocities from U_{GS} = 5 m/s to the actual critical gas velocity needed to generate large-amplitude waves. The correlation does a good job for very viscous liquids at gas velocities larger than $U_{GS,t}$.

Design procedure

An iterative procedure similar to what has been suggested by other investigators (Russel et al., 1974; Cheremisinoff and Dav-

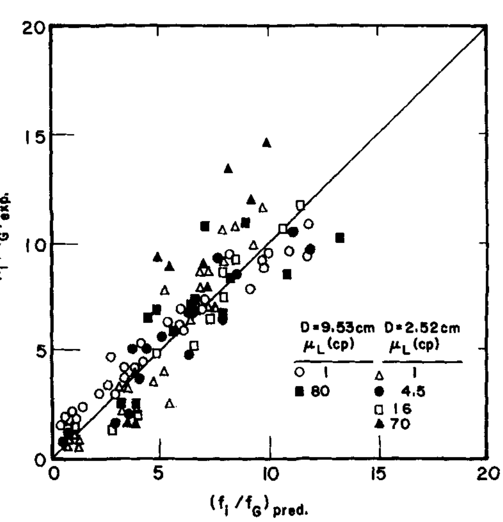

Figure 14. Comparison of experimental values of f_i/f_G with those predicted by Eq. 29.

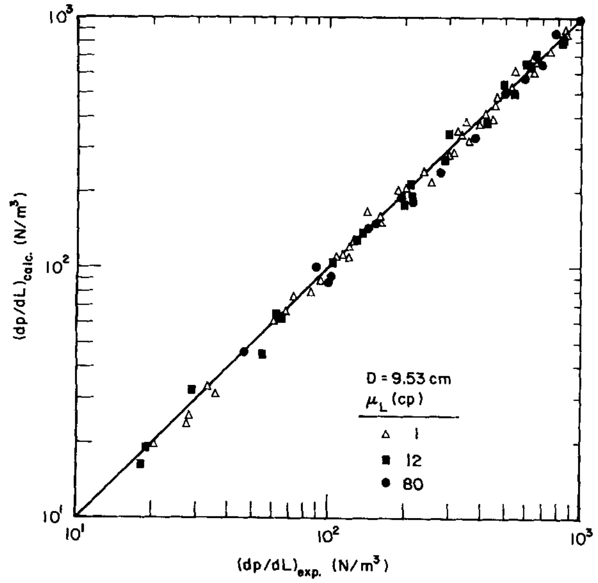

Figure 15. Comparison of calculated pressure drops with data in 9.53 cm dia. pipe.

is, 1979) can be used to calculate frictional pressure losses and h/D (or holdup) for stratified flows in horizontal pipelines.

1. Assume a value of h/D and calculate the geometric variables, the actual gas and liquid velocities, and the gas and liquid Reynolds numbers.

2. The pressure drop is calculated from Eq. 1 using the following equations:

For the gas friction factor

$$f_G = 0.046\,Re_G^{-0.2} \quad \text{for } Re_G > 2{,}000 \tag{10}$$

$$f_G = 16/Re_G \quad \text{for } Re_G < 2{,}000 \tag{31}$$

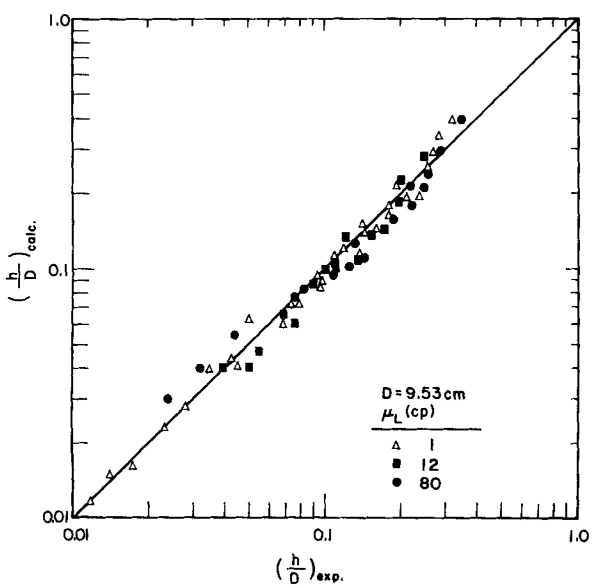

Figure 16. Comparison of calculated film thicknesses with data in 9.53 cm dia. pipe.

For the interfacial friction factor

$$\frac{f_i}{f_G} = 1 \quad \text{for } U_{GS} \le \left(\frac{\rho_{GO}}{\rho_G}\right)^{0.5} (5\ \text{m/s})$$

$$\frac{f_i}{f_G} = 1 + 15\left(\frac{h}{D}\right)^{0.5}\left[\frac{U_{GS}}{5}\left(\frac{\rho_G}{\rho_{GO}}\right)^{0.5} - 1\right]$$

$$\text{for } U_{GS} \ge \left(\frac{\rho_{GO}}{\rho_G}\right)^{0.5} (5\ \text{m/s})$$

where U_{GS} is in units of m/s.

3. Calculate the dimensionless liquid film height h^+ from Eq. 23, the characteristic stress from the equation

$$\tau_c = \rho_L \left(\frac{h^+ \nu_L}{D}\right)^2 \left(\frac{h}{D}\right)^{-2} \tag{32}$$

and the liquid wall shear stress τ_{WL} from Eq. 12.

4. Calculate the pressure drop in the liquid phase, using Eq. 2. If this is not the same as calculated in Eq. 1 for the gas phase, a new value of h/D is assumed.

The iterative procedure is simplified if the Blasius equation or Poiseuille's law is used to evaluate τ_{WL} (a modified Taitel-Dukler approach): A value of f_i/f_G is assumed; Figures 2 and 3 are used to calculate h/D and the frictional pressure drop; a new value of f_i/f_G is calculated with the equations presented above, and the procedure is repeated.

Comparison with measurements

Experimental results obtained for studies in the 9.53 cm dia. pipe are compared with predictions in Figures 15 and 16. Data at low gas velocities ($U_{GS} < 3$ m/s), where the flow is not fully developed, and at high gas velocities ($U_{GS} > 22$ m/s), where the liquid starts to climb up the walls of the pipe, are not included. Similar agreement is obtained between the predictions and measurements in the 2.52 cm dia. pipe.

A comparison between experimental liquid heights and values predicted from the Taitel-Dukler (T-D) model ($f_i = f_G$), the modified T-D model (Figures 2 and 3), and the procedure recommended in this paper is given in Figure 17. At low gas velocities the T-D model and the modified T-D model give the same

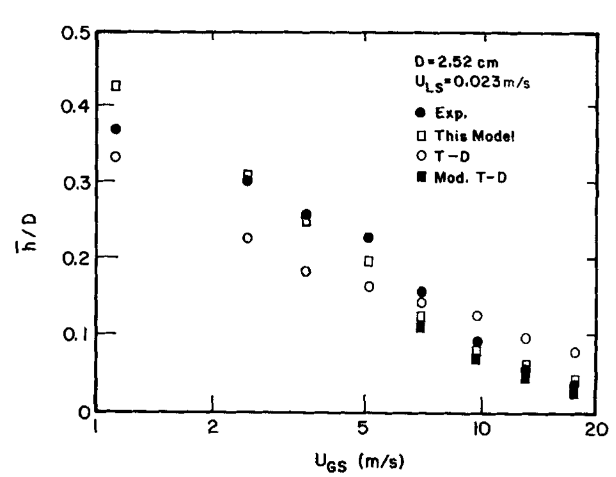

Figure 17. Comparison between experimental and calculated values of h/D.

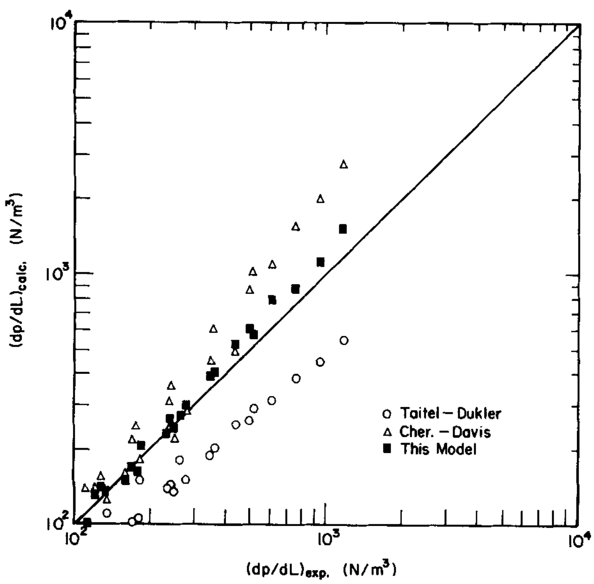

Figure 18. Comparison of data of Hoogendoorn with various models.

results: both underpredict the liquid height. On the other hand, the modified T-D model gives a satisfactory prediction at high gas velocities.

Figure 18 compares the data of Hoogendoorn (1959) for a 2.36 cp (mPa · s) oil in a 14 cm dia. pipe with various design procedures. The agreement between the data and the proposal in this paper is good, particularly considering that errors of ±10% are possible in getting numbers from Figure 9 of Hoogendoorn's paper. The Cheremisinoff-Davis method predicts values of frictional pressure drop that are too high and the T-D method give values that are too small.

The modified T-D approach gives a good agreement since, for these experiments, the liquid Reynolds numbers are very large.

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Notation

A = area
C = wave velocity
D = diameter or hydraulic diameter
D_L = hydraulic diameter = $4A_L/P_L$
D_G = hydraulic diameter = $4A_G/(P_G + S_i)$
f = friction factor
g = gravitational acceleration constant
h = height of liquid film
Δh = amplitude of wave
k = wave number
k_m = characteristic wave number
p = pressure
P = perimeter
r, R = radius
Re_L = Reynolds number of liquid = $D_L U_L/\nu_L$
Re_G = Reynolds number of gas = $D_G U_G/\nu_G$
S_i = width of interface
u = local velocity of liquid phase
u^* = friction velocity
u_c^* = friction velocity, Eq. 13
U_{GS}, U_{LS} = superficial velocities of gas and liquid
U = actual bulk velocity
x = coordinate in direction of flow
y = coordinate perpendicular to direction of flow

Greek letters

γ = angle, Figure 1
δ = length, Figure 1
Δh = wave amplitude
κ = von Karman
ϵ = eddy viscosity
θ = angle, Figure 1
λ = wavelength
μ = viscosity
ν = kinematic viscosity
ρ = density
τ = shear stress
τ_c = characteristic stress, Eq. 12
Φ = Lockhart-Martinelli parameter
χ = Lockhart-Martinelli parameter

Subscripts

C = characteristic
G = gas phase
i = at gas-liquid interface
L = liquid phase
O = at atmospheric pressure
t = transition
W = at wall

Superscript

$+$ = quantity made dimensionless using v_c^* and ν_L

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